






## Letter

## Long Duration Coverage Control of Multiple Robotic Surface Vehicles Under Battery Energy Constraints

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Dear Editor,

This letter addresses long duration coverage problem of multiple robotic surface vehicles (RSVs) subject to battery energy constraints, in addition to uncertainties and disturbances. An anti-disturbance energy-aware control method is proposed for performing coverage task of RSVs. Firstly, a centroidal Voronoi tessellation (CVT) is used to optimize the partition of the given coverage area. The optimal position for each vehicle corresponds to the centroid of the Voronoi cell. Secondly, by consisting two battery energy control barrier functions, an energy-aware kinematic guidance law is designed to drive each RSV to the optimal position. Finally, an anti-disturbance fixed-time kinetic control law is designed for each RSV to track the desired speed based on the fixed-time extended state observer and nonlinear tracking differentiator. By the control method, RSVs are capable of achieving the long duration coverage within the given coverage area. Simulation results verify the effectiveness of the proposed anti-disturbance energy-aware control method for multi-RSV.

There is a surge of interest in motion control of RSVs because they are vital tools in exploring the oceans [1]–[3]. Cooperative control enables RSVs to perform more challenging task over a single RSV in terms of enhanced efficiency and effectiveness [4], [5]. In particular, coverage control is a typical cooperative control of RSVs. Such motion control scenario can find numerous missions in practice such as maritime patrol and environment monitoring [1], [6]–[9]. Coverage control is proposed for unmanned ground vehicles [6], unmanned aerial vehicles [7], and robotic surface vehicles [8]. However, the aforementioned works [6]–[9] do not explicitly guarantee the persistent coverage performance. In practice, the vehicles may not execute task persistently because of the battery energy constraints.

Motivated by the above observations, this letter aims to design an anti-disturbance energy-aware control method for RSVs to execute long duration coverage task. The main contributions of this letter can be summarized as: 1) In contrast to the works in [4], [5] where the objectives are to maintain formation, the proposed control method for

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multi-RSV to achieve coverage task within a given coverage area. 2) In contrast to the existing works [6]–[9] where the persistent coverage performance may not be guaranteed definitely, an energy-aware control method based on two battery energy control barrier functions (BE-CBFs) are designed to achieve long duration coverage mission. 3) In contrast to the existing coverage control methods in [8], [9] where the convergence time of estimation errors can not be calculated in a prescribed time, a fixed time extended state observer (FTESO) is proposed such that the estimation errors are convergent to zero within a fixed time.

**Problem formulation:** Consider the dynamics of RSVs as

$$\begin{cases} \dot{x}_i = u_i \cos(\psi_{bi}) - v_i \sin(\psi_{bi}) \\ \dot{y}_i = u_i \sin(\psi_{bi}) + v_i \cos(\psi_{bi}) \\ \dot{\psi}_{bi} = r_i \\ m_{ui} \dot{u}_i = f_{ui}(u_i, v_i, r_i) + \tau_{ui} + \tau_{wui} \\ m_{vi} \dot{v}_i = f_{vi}(u_i, v_i, r_i) + \tau_{vvi} \\ m_{ri} \dot{r}_i = f_{ri}(u_i, v_i, r_i) + \tau_{ri} + \tau_{wri} \end{cases} \quad (1)$$

where  $x_i$ ,  $y_i$ , and  $\psi_{bi}$  are the position and orientation in an earth-fixed inertial frame;  $u_i$ ,  $v_i$ , and  $r_i$  are the velocities in the surge, sway, and yaw directions respectively in a body-fixed frame;  $m_{ui}$ ,  $m_{vi}$ , and  $m_{ri}$  are the mass of RSVs;  $f_{ui}(\cdot)$ ,  $f_{vi}(\cdot)$ , and  $f_{ri}(\cdot)$  represent unknown nonlinear functions;  $\tau_{ui}$  and  $\tau_{ri}$  denote the surge force and yaw moment;  $\tau_{wui}$ ,  $\tau_{vvi}$ , and  $\tau_{wri}$  denote the time-varying environmental disturbances. Denote  $U_i = \sqrt{u_i^2 + v_i^2}$  as the total speed of the  $i$ th RSV, and the dynamics (1) of the RSV can be rewritten as

$$\begin{cases} \dot{x}_i = U_i \cos(\psi_i), \quad \dot{y}_i = U_i \sin(\psi_i), \quad \dot{\psi}_i = r_i + \dot{\beta}_i \\ m_{ui} \dot{U}_i = \cos(\beta_i)(f_{ui} + \tau_{wui}) - 2 \sin^2\left(\frac{\beta_i}{2}\right) \tau_{ui} \\ \quad + \sin(\beta_i)(f_{vi} + \tau_{vvi}) m_{ui} / m_{vi} + \tau_{ui} \\ m_{ri} \dot{r}_i = f_{ri} + \tau_{ri} + \tau_{wri} \end{cases} \quad (2)$$

where  $\psi_i = \psi_{bi} + \beta_i$  is the course angle with  $\beta_i = \text{atan2}(v_i, u_i)$ .

The charging and discharging model of the battery is stated as  $\dot{E}_i = -\delta K_{ch} + (1 - \delta) K_{en}$ , where  $E_i$  is the battery energy level of the  $i$ th RSV;  $K_{ch} > 0 / K_{en} > 0$  represents the case of the worst/fastest battery discharging/charging;  $\delta \in [0, 1]$  is the switching coefficient of battery charging and discharging.

**Design and analysis:** Step 1: The first step is optimizing the partition of the given coverage area. CVT is a powerful tool for the area partition. The centroid of each Voronoi cell can be defined as the optimal position for each RSV to track. Consider that  $N$  RSVs are randomly distributed in a closed and connected area  $G \in \mathbb{R}^2$  that has to be covered. Associate a density function  $\phi(g) : g \in \mathbb{R}^2$  with each point  $g \in G$ . Define locational cost function of the multi-RSV as  $H(P) = \sum_{i=1}^N \int_{V_i} \|p_i - g\|^2 \phi(g) dg$ , where  $p_i = [x_i, y_i]^T$  is the position of  $i$ th RSV;  $P = [p_1, \dots, p_N]$  is position set of RSVs;  $\|p_i - g\|$  denotes the Euclidean distance metric. Next, the Voronoi cell of the  $i$ th RSV can be defined as  $V_i = \{g \in G \mid \|g - p_i\| \leq \|g - p_j\|, \forall i \neq j\}$ . Then, it follows from [8] that the optimal position  $p_{ci}$  corresponds to the centroid of the Voronoi cell in a CVT is:

$$p_{ci} = \frac{\int_{V_i} g \phi(g) dg}{M_{V_i}} \quad (3)$$

where  $M_{V_i}$  is the mass of the  $V_i$  with  $M_{V_i} = \int_{V_i} \phi(g) dg$ .

Step 2: Define the target tracking error as  $e_{pi} = p_i - p_{ci}$ . Take the derivative of  $e_{pi}$  along (2), and one has  $\dot{e}_{pi} = q_i - \dot{p}_{ci}$ , where

$q_i = [q_{xi}, q_{yi}]^T = U_i[\cos(\psi_{bi}), \sin(\psi_{bi})]^T$ . To stabilize  $e_{pi}$ , an optimal total speed guidance law is designed as  $q_{ci} = -K_{qi}e_{pi}/\sqrt{\|e_{pi}\|^2 + \Delta_{qi}^2} + \dot{p}_{ci}$ , where  $K_{qi} \in \mathbb{R}^2$  is the guidance law parameter vector and  $\Delta_{qi}$  is a positive constant. Define the velocity tracking error as  $e_{qi} = q_i - q_{ci}$  and the time derivative of the  $e_{pi}$  can be rewritten

$$\dot{e}_{pi} = -K_{qi}e_{pi}/\sqrt{\|e_{pi}\|^2 + \Delta_{qi}^2} + e_{qi}. \quad (4)$$

To implement the long duration coverage control task, the position of charging station  $p_{bi}$  should satisfy  $E_{\text{charge}} = \{\varepsilon_i(p_i) \in \mathbb{R}^2 \mid \|p_i - p_{bi}\| \leq d_{ch}\}$ , where  $\varepsilon_i(p_i)$  is the minimum energy required for the  $i$ th RSV to return to the charging station and  $d_{ch}$  is the radius of the charging station. In order to maintain a desired energy reserve, the following inequalities  $E_i - E_{\min} > \varepsilon_i(p_i)$  and  $E_{\max} - E_i > 0$  hold, where  $E_{\min}$  denotes the minimum capacity of battery storage and  $E_{\max}$  is the maximum capacity of battery storage. It means that each RSV can return to the charging station before limited battery energy depletion and leave the charging station before overcharging. According to the definition of the CBF, the functions  $h_{en}$  and  $h_{cn}$  are defined as  $h_{en}(z_i) = E_i - E_{\min} - \varepsilon_i(p_i)$  and  $h_{cn}(z_i) = E_{\max} - E_i - \delta(s_{i0}) - E_i$ , where  $z_i = [p_i^T, E_i]^T$ ,  $s_{i0} = \|p_i - p_{i0}\|$ , and  $\delta(s_{i0})$  is a step function. If  $s_{i0} > d_{ch}$ ,  $\delta(s_{i0}) = 1$ ; otherwise,  $\delta(s_{i0}) = 0$ . The safe set  $C_{zi}$  in this letter is defined as  $C_{zi} = \{z_i \mid h_{en}(z_i) \geq 0, h_{cn}(z_i) \geq 0\}$ .

It follows from the safe set that the functions  $h_{en}$  and  $h_{cn}$  can be described by an equivalent  $h(z_i)$  as  $h(z_i) = \min\{h_{en}(z_i), h_{cn}(z_i)\}$ . Thus, the speed guidance law  $q_{ci}^* = [q_{xi}^*, q_{yi}^*]^T$  can be obtained by the following quadratic programming problem:

$$\begin{aligned} \arg \min_{q_{ci}^* \in \mathbb{R}^2} J_i(q_{ci}^*) &= \|q_i - q_{ci}^*\|^2 \\ \text{s.t. } \dot{h}(z_i) &\geq -\alpha h(z_i) \end{aligned} \quad (5)$$

where  $\dot{h}(z_i) = (\frac{\partial h}{\partial p_i})^T q_{ci}^* - \delta K_{ch} + (1 - \delta)K_{en}$ . The optimal total speed of the  $i$ th RSV is derived as  $U_{ci} = q_{xi}^* \cos(\psi_i) + q_{yi}^* \sin(\psi_i)$ .

Step 3: Define  $\psi_{ci} = \text{atan2}(q_{yi}^*, q_{xi}^*)$  and  $e_{\psi i} = \psi_i - \psi_{ci}$ , and the dynamics of  $e_{\psi i}$  is  $\dot{e}_{\psi i} = r_i + \beta_i - \dot{\psi}_{ci}$ . To stabilize  $e_{\psi i}$ , a yaw guidance law is designed as

$$r_{ci} = -k_{ri}e_{\psi i}/\sqrt{\|e_{\psi i}\|^2 + \Delta_{ri}^2} - \dot{\beta}_i + \dot{\psi}_{ci} \quad (6)$$

where  $k_{ri} \in \mathbb{R}$  is the guidance law parameter and  $\Delta_{ri}$  is a positive constant. Define  $e_{ri} = r_i - r_{ci}$ , and the dynamics of the yaw target tracking error is rewritten as

$$\dot{e}_{ri} = -k_{ri}e_{\psi i}/\sqrt{\|e_{\psi i}\|^2 + \Delta_{ri}^2} + e_{ri}. \quad (7)$$

Step 4: Due to the fact that the uncertainties and disturbances can affect the performance of the control system seriously [10]–[12], it is necessary to recover the uncertainties and disturbances. First, to obtain a smoother motion profile for RSV, let  $U_{ci}$  and  $r_{ci}$  pass through two nonlinear tracking differentiators as

$$\begin{cases} \dot{\hat{U}}_{ci} = U_{di} - k_{i1}^U \text{sig}^{1-1/k_{i3}^U}(\hat{U}_{ci} - U_{ci}) \\ \dot{\hat{U}}_{di} = -k_{i2}^U \text{sig}^{1-2/k_{i3}^U}(\hat{U}_{ci} - U_{ci}) \\ \dot{\hat{r}}_{ci} = r_{di} - k_{i1}^r \text{sig}^{1-1/k_{i3}^r}(\hat{r}_{ci} - r_{ci}) \\ \dot{\hat{r}}_{di} = -k_{i2}^r \text{sig}^{1-2/k_{i3}^r}(\hat{r}_{ci} - r_{ci}) \end{cases} \quad (8)$$

where  $\hat{U}_{ci}$  is the estimate of the  $U_{ci}$ ;  $\hat{r}_{ci}$  is the estimate of the  $r_{ci}$ ;  $U_{di}$  is the estimate of the  $\dot{U}_{ci}$ , and  $r_{di}$  is the estimate of the  $\dot{r}_{ci}$ .  $k_{i1}^U \in \mathbb{R}^+$ ,  $k_{i2}^U \in \mathbb{R}^+$ ,  $k_{i3}^U > 2$ ,  $k_{i1}^r \in \mathbb{R}^+$ ,  $k_{i2}^r \in \mathbb{R}^+$ , and  $k_{i3}^r > 2$  are the design parameters. Define  $e_{Ui}^c = \hat{U}_{ci} - U_{ci}$ ,  $e_{ri}^c = \hat{r}_{ci} - r_{ci}$ ,  $e_{Ui}^d = U_{di} - \hat{U}_{ci}$ , and  $e_{ri}^d = r_{di} - \hat{r}_{ci}$ . It follows from (8) that the dynamics of the  $e_{Ui}^c$ ,  $e_{ri}^c$ ,  $e_{Ui}^d$ , and  $e_{ri}^d$  are obtained as:

$$\begin{cases} \dot{e}_{Ui}^c = e_{Ui}^d - k_{i1}^U \text{sig}^{1-1/k_{i3}^U} e_{Ui}^c \\ \dot{e}_{Ui}^d = -k_{i2}^U \text{sig}^{1-2/k_{i3}^U} e_{Ui}^c U_{di} - \dot{U}_{ci} \\ \dot{e}_{ri}^c = e_{ri}^d - k_{i1}^r \text{sig}^{1-1/k_{i3}^r} e_{ri}^c \\ \dot{e}_{ri}^d = -k_{i2}^r \text{sig}^{1-2/k_{i3}^r} e_{ri}^c r_{di} - \dot{r}_{ci}. \end{cases} \quad (9)$$

From (2), the kinetics of the  $i$ th RSV can be rewritten as  $\dot{U}_i = g_{ui} + m_{ui}^{-1}\tau_{ui}$  and  $\dot{r}_i = g_{ri} + m_{ri}^{-1}\tau_{ri}$ , where  $g_{ui} = m_{ui}^{-1}(\cos(\beta_i)(f_{ui} + \tau_{wui}) - 2\sin^2(\frac{\beta_i}{2})\tau_{ui} + \sin(\beta_i)(f_{vi} + \tau_{wvi}))m_{ui}/m_{vi}$  and  $g_{ri} = m_{ri}^{-1}(f_{ri} + \tau_{wri})$ . Two fixed-time ESOs are designed to estimate the unknown functions  $g_{ui}$  and  $g_{ri}$

$$\begin{aligned} \dot{\hat{U}}_i &= -k_{i1}^u [\tilde{U}_i]^{\alpha_{ui}} - k_{i2}^u [\tilde{U}_i]^{\beta_{ui}} + \hat{g}_{ui} + m_{ui}^{-1}\tau_{ui} \\ \dot{\hat{g}}_{ui} &= -k_{i3}^u [\tilde{U}_i]^{2\alpha_{ui}-1} - k_{i4}^u [\tilde{U}_i]^{2\beta_{ui}-1} + k_{i0}^u \text{sign}(\tilde{U}_i) \\ \dot{\hat{r}}_i &= -k_{i1}^r [\tilde{r}_i]^{\alpha_{ri}} - k_{i2}^r [\tilde{r}_i]^{\beta_{ri}} + \hat{g}_{ri} + m_{ri}^{-1}\tau_{ri} \\ \dot{\hat{g}}_{ri} &= -k_{i3}^r [\tilde{r}_i]^{2\alpha_{ri}-1} - k_{i4}^r [\tilde{r}_i]^{2\beta_{ri}-1} + k_{i0}^r \text{sign}(\tilde{r}_i) \end{aligned} \quad (10)$$

where  $k_{ia}^u \in \mathbb{R}^+$  and  $k_{ia}^r \in \mathbb{R}^+$  ( $a = 1, 2, 3, 4$ ) are the observer gains;  $\hat{U}_i$ ,  $\hat{r}_i$ ,  $\hat{g}_{ui}$ , and  $\hat{g}_{ri}$  are the estimates of  $U_i$ ,  $r_i$ ,  $g_{ui}$ , and  $g_{ri}$  respectively;  $\tilde{U}_i = \hat{U}_i - U_i$ ,  $\tilde{r}_i = \hat{r}_i - r_i$ ,  $\alpha_{ui} \in (1 - \varepsilon_{ui}, 1)$ ,  $\beta_{ui} = 1/\alpha_{ui}$ ,  $\alpha_{ri} \in (1 - \varepsilon_{ri}, 1)$ , and  $\beta_{ri} = 1/\alpha_{ri}$  with  $\varepsilon_{ui}$  and  $\varepsilon_{ri}$  being the small constants;  $k_{i0}^u > 0$  and  $k_{i0}^r > 0$ .

Assumption 1: For unknown functions  $g_{ui}$  and  $g_{ri}$ , there are  $g_{ui}^* \in \mathbb{R}^+$  and  $g_{ri}^* \in \mathbb{R}^+$  such that  $|g_{ui}| \leq g_{ui}^*$  and  $|g_{ri}| \leq g_{ri}^*$ .

Step 5: Let  $\tilde{g}_{ui} = \hat{g}_{ui} - g_{ui}$  and  $\tilde{g}_{ri} = \hat{g}_{ri} - g_{ri}$ . It follows from equation (10) that the error dynamics can be expressed as:

$$\begin{aligned} \dot{\tilde{U}}_i &= -k_{i1}^u [\tilde{U}_i]^{\alpha_{ui}} - k_{i2}^u [\tilde{U}_i]^{\beta_{ui}} + \tilde{g}_{ui} \\ \dot{\tilde{g}}_{ui} &= -k_{i3}^u [\tilde{U}_i]^{2\alpha_{ui}-1} - k_{i4}^u [\tilde{U}_i]^{2\beta_{ui}-1} - \tilde{g}_{ui} + k_{i0}^u \text{sign}(\tilde{U}_i) \\ \dot{\tilde{r}}_i &= -k_{i1}^r [\tilde{r}_i]^{\alpha_{ri}} - k_{i2}^r [\tilde{r}_i]^{\beta_{ri}} + \tilde{g}_{ri} \\ \dot{\tilde{g}}_{ri} &= -k_{i3}^r [\tilde{r}_i]^{2\alpha_{ri}-1} - k_{i4}^r [\tilde{r}_i]^{2\beta_{ri}-1} - \tilde{g}_{ri} + k_{i0}^r \text{sign}(\tilde{r}_i). \end{aligned} \quad (11)$$

Define the tracking errors as  $e_{Ui} = U_i - \hat{U}_{ci}$  and  $e_{ri} = r_i - \hat{r}_{ci}$ . Taking the derivatives of  $e_{Ui}$  and  $e_{ri}$  yields

$$\begin{cases} \dot{e}_{Ui} = g_{ui} + m_{ui}^{-1}\tau_{ui} - \dot{\hat{U}}_{ci} \\ \dot{e}_{ri} = g_{ri} + m_{ri}^{-1}\tau_{ri} - \dot{\hat{r}}_{ci}. \end{cases} \quad (12)$$

Two anti-disturbance fixed-time kinetic control laws are designed

$$\begin{cases} \tau_{ui} = m_{ui}(-k_{i1}^{\tau} [e_{Ui}]^{\alpha_{ui}} - k_{i2}^{\tau} [e_{Ui}]^{\beta_{ui}} - \hat{g}_{ui} + \dot{\hat{U}}_{ci}) \\ \tau_{ri} = m_{ri}(-k_{i3}^{\tau} [e_{ri}]^{\alpha_{ri}} - k_{i4}^{\tau} [e_{ri}]^{\beta_{ri}} - \hat{g}_{ri} + \dot{\hat{r}}_{ci}) \end{cases} \quad (13)$$

where  $k_{ia}^{\tau} \in \mathbb{R}^+$  ( $a = 1, 2, 3, 4$ ) are the control gains. Then, the tracking error systems (12) can be put into  $\dot{e}_{Ui} = -k_{i1}^{\tau} [e_{Ui}]^{\alpha_{ui}} - k_{i2}^{\tau} [e_{Ui}]^{\beta_{ui}} - \tilde{g}_{ui}$  and  $\dot{e}_{ri} = -k_{i3}^{\tau} [e_{ri}]^{\alpha_{ri}} - k_{i4}^{\tau} [e_{ri}]^{\beta_{ri}} - \tilde{g}_{ri}$ .

Lemma 1: The tracking error subsystem consisting of (4) and (7):  $[e_{qi}, e_{ri}] \mapsto [e_{pi}, e_{\psi i}]$  is input-to-state stable (ISS).

Proof: Choose the Lyapunov function as  $V_{i1}^e = \sum_{i=1}^N (1/2)(e_{pi}^T e_{pi} + e_{\psi i}^2)$ . The time derivative of  $V_{i1}^e$  is  $\dot{V}_{i1}^e \leq \sum_{i=1}^N (-\lambda_{\max}(K_{i1})\|H_{i1}\|^2 / \sqrt{\|H_{i1}\|^2 + \delta_{i\max}} + \|h_{i1}\| \|H_{i1}\|)$ , where  $H_{i1} = [e_{pi}^T, e_{\psi i}]$ ,  $K_{i1} = \text{diag}\{K_{qi}, K_{ri}\}$ ,  $\delta_{i\max} = \max\{\delta_{iq}, \delta_{ir}\}$ ,  $h_{i1} = [e_{qi}, e_{ri}]$ , and  $\lambda_{\min}(K_{i1})$  is the minimum eigenvalue of a square matrix  $K_{i1}$ . Note that as  $\|H_{i1}\|^2 / \sqrt{\|H_{i1}\|^2 + \delta_{i\max}} \geq \|h_{i1}\| / (a_{i1} \lambda_{\min}(K_{i1}))$ ,  $\dot{V}_{i1}^e \leq -\sum_{i=1}^N \{\lambda_{\min}(K_{i1})(1 - a_{i1})\}$ . ■

Lemma 2: Under Assumption 1, with the FTESO (10), the states  $\hat{U}_i$ ,  $\hat{g}_{ui}$ ,  $\hat{r}_i$ , and  $\hat{g}_{ri}$  can achieve the estimations in a fixed time.

Proof: Take the following part from error dynamics (11):

$$\begin{cases} \dot{\tilde{U}}_i = -k_{i1}^u [\tilde{U}_i]^{\alpha_{ui}} + \tilde{g}_{ui} \\ \dot{\tilde{g}}_{ui} = -k_{i3}^u [\tilde{U}_i]^{2\alpha_{ui}-1}. \end{cases} \quad (14)$$

Define  $F_{i1} = [\tilde{U}_i, \tilde{g}_{ui}]^T$  and  $\zeta_{i1} = [\tilde{U}_i, ((\tilde{g}_{ui})^{1/\alpha_{ui}})]^T$ . Consider a Lya-

punov function  $V_{i1}^u(\zeta_{i1}) = \zeta_{i1}^T P_{i1} \zeta_{i1}$ , where  $P_{i1}$  is a symmetric positive definite matrix satisfying  $P_{i1} A_{i1} + A_{i1}^T P_{i1} = -I$  and  $A_{i1}$  is a Hurwitz matrix with  $A_{i1} = [-k_{i1}^u, 1; -k_{i3}^u, 0]$ . Setting  $\alpha_{ui} = 1$ , the equation (14) becomes  $\dot{F}_{i1} = A_{i1} F_{i1}$ . Taking the time derivatives of  $V_{i1}^u(F_{i1})$  gives  $\dot{V}_{i1}^u(F_{i1}) = F_{i1}^T (P_{i1} A_{i1} + A_{i1}^T P_{i1}) F_{i1} = -F_{i1}^T F_{i1} \leq 0$ . Define  $\sigma_{ui}$  as a small constant, and (14) is locally asymptotically stable by choosing  $\alpha_{ui}$  in the interval  $(1 - \sigma_{ui}, 1)$ . Then, from Lemma 2 in [5],  $V_{i1}^u(\zeta_{i1})$  is homogeneous of degree  $\varrho_{i1}$  with  $(0, \alpha_{ui})$  and its derivative  $\dot{V}_{i1}^u(\zeta_{i1})$  is homogeneous of degree  $\varrho_{i1} + \alpha_{ui} - 1$ . One has  $V_{i1}^u(F_{i1}) \leq \lambda_{\max}(P_{i1}) \|F_{i1}\|^2$  and  $\dot{V}_{i1}^u(F_{i1}) \leq -\|F_{i1}\|^2$ . It renders that  $\dot{V}_{i1}^u(F_{i1}) \leq -V_{i1}^u(F_{i1}) / \lambda_{\max}(P_{i1})$ . The second part of (11) is expressed as  $\dot{U}_i = -k_{i2}^u [\tilde{U}_i]^{\beta_{ui}} + \tilde{g}_{ui}$  and  $\dot{\tilde{g}}_{ui} = -k_{i4}^u [\tilde{U}_i]^{2\beta_{ui}-1}$ . Define  $\zeta_{i2} = [\tilde{U}_i, ((\tilde{g}_{ui})^{1/\beta_{ui}})]^T$ . Consider a Lyapunov function  $V_{i2}(\zeta_{i2}) = \zeta_{i2}^T P_{i2} \zeta_{i2}$ . Similar to the analysis in (14), one has  $\dot{V}_{i2}(\zeta_{i2}) \leq -V_{i2}(\zeta_{i2})^{\varrho_{i2} + \beta_{ui} - 1} / \varrho_{i2} / \lambda_{\max}(P_{i2})$ . For

$$\begin{cases} \dot{\tilde{U}}_i = -k_{i1}^u [\tilde{U}_i]^{\alpha_{ui}} - k_{i2}^u [\tilde{U}_i]^{\beta_{ui}} + \tilde{g}_{ui} \\ \dot{\tilde{g}}_{ui} = -k_{i3}^u [\tilde{U}_i]^{2\alpha_{ui}-1} - k_{i4}^u [\tilde{U}_i]^{2\beta_{ui}-1} \end{cases} \quad (15)$$

there exists  $\iota_{ui}$  such that  $V_{i2}^u(\zeta_{i2}(t_0)) > \iota_{ui}$  with  $\iota_{ui}$  being  $0 < \iota_{ui} < \lambda_{\min}(P_{i2})$ . It renders that  $V_{i2}^u$  reaches  $\iota_{ui}$  within a fixed-time  $T_{i1}^u = \varrho_{i2} \lambda_{\max}(P_{i2}) / (\beta_{ui} - 1) \iota_{ui}^{(\beta_{ui}-1)/\varrho_{i2}}$ . Besides, the inequalities  $\|\zeta_{i2}\|^2 \leq V_{i2}^u(\zeta_{i2}) / \lambda_{\min}(P_{i2}) \leq \iota_{ui} / \lambda_{\min}(P_{i2}) \leq 1$  and  $V_{i1}^u(\zeta_{i1}) \leq \lambda_{\max}(P_{i1}) \|\zeta_{i1}\|^2$  hold. If  $V_{i1}^u(\zeta_{i1}) < \iota_{ui}$  and  $\|\zeta_{i1}\| = 1$ ,  $\|\zeta_{i2}\|^2 \leq \|\zeta_{i1}\|^2 \leq 1$  and  $V_{i1}(\zeta_{i1}) \leq \lambda_{\max}(P_{i1})$ . The settling time of the system (14) satisfies  $T_{i2}^u \leq \varrho_{i1} \lambda_{\max}^{(\varrho_{i1} + \alpha_{ui} - 1) / \varrho_{i1}}(P_{i1}) / (1 - \alpha_{ui})$ . As a result, the states of the system (15) are convergent to zero within a fixed time  $T_{i1}^u + T_{i2}^u$ . If  $t \geq T_{i1}^u + T_{i2}^u$ , the system (11) becomes  $\dot{\tilde{g}}_{ui} = k_{i0}^u \text{sgn}(\tilde{U}_i) - \tilde{g}_{ui}$ . Consider a Lyapunov function  $V_{i3}^u = (1/2)(\tilde{g}_{ui})^2$ . The derivative of  $V_{i3}^u$  can be obtained as  $\dot{V}_{i3}^u \leq -(k_{i0}^u - g_{ui}^*) \sqrt{2V_{i3}^u}$ . In summary,  $\tilde{g}_{ui}$  is convergent to zero within the time  $T_{i3}^u \leq \sqrt{2V_{i3}^u(T_{i1}^u + T_{i2}^u)} / (k_{i0}^u - g_{ui}^*)$ . Thus, the states  $\tilde{U}_i$  and  $\tilde{g}_{ui}$  are convergent to zero within a fixed time  $T_{ui} \leq T_{i1}^u + T_{i2}^u + T_{i3}^u$  regardless of the initial conditions. Similarly, the states  $\tilde{r}_i$  and  $\tilde{g}_{ri}$  achieve the estimations within the fixed time  $T_{ri} \leq T_{i1}^r + T_{i2}^r + T_{i3}^r$  with  $T_{i1}^r = \varrho_{i4} \lambda_{\max}(P_{i4}) / (\beta_{ri} - 1) \iota_{ri}^{(\beta_{ri}-1)/\varrho_{i4}}$ ,  $T_{i2}^r \leq \varrho_{i3} \lambda_{\max}^{(\varrho_{i3} + \alpha_{ri} - 1) / \varrho_{i3}}(P_{i3}) / (1 - \alpha_{ri})$ , and  $T_{i3}^r \leq \sqrt{2V_{i3}^r(T_{i1}^r + T_{i2}^r)} / (k_{i0}^r - g_{ri}^*)$  regardless of the initial conditions. The definitions of  $P_{i2}$ ,  $P_{i3}$ ,  $P_{i4}$ ,  $A_{i2}$ ,  $A_{i3}$ ,  $A_{i4}$ ,  $\varrho_{i2}$ ,  $\varrho_{i3}$ ,  $\varrho_{i4}$ , and  $\iota_{ri}$  analogize the definitions of  $P_{i1}$ ,  $A_{i1}$ ,  $\varrho_{i1}$ , and  $\iota_{ui}$ . ■

**Theorem 1:** Under Assumption 1, by using the optimal total speed guidance law (5), the yaw guidance law (6), the nonlinear tracking differentiator (8), the fixed-time ESOs (10), and the anti-disturbance kinetic fixed-time control laws (12), the RSVs described by (2) are able to cover the centroid of the Voronoi cell (3) at any initial states with the safe set  $C_{zj}$ . Besides, all the errors in the closed-loop system are uniformly ultimately bounded.

**Proof:** According to Lemma 1, the tracking errors  $e_{qi}$ ,  $e_{pi}$ , and  $e_{\psi i}$  are uniformly ultimately bounded (UUB). By [5], it can obtain that the errors  $e_{U_i}^c$ ,  $e_{r_i}^c$ ,  $e_{U_i}^d$ , and  $e_{r_i}^d$  can converge to a small neighborhood of the origin. Construct the Lyapunov function as  $V_{i4} = (1/2)(e_{U_i}^2 + e_{r_i}^2)$ . Taking the time derivative of the  $V_{i4}$  along (12), it can be obtained that  $\dot{V}_{i4} \leq -2^{(1+\alpha_{ui}^r)/2} k_{i1}^r (1/2|e_{U_i}|^2)^{1+\alpha_{ui}^r} - 2^{(1+\beta_{ui}^r)/2} k_{i2}^r \times (1/2|e_{U_i}|^2)^{1+\beta_{ui}^r} - 2^{(1+\alpha_{ri}^r)/2} k_{i3}^r ((1/2)|e_{r_i}|^2)^{1+\alpha_{ri}^r} - 2^{(1+\beta_{ri}^r)/2} k_{i4}^r ((1/2)|e_{r_i}|^2)^{1+\beta_{ri}^r} + g_{ui}^* |e_{U_i}| + g_{ri}^* |e_{r_i}|$ . Then, the following inequality holds  $\dot{V}_{i4} \leq -k_{i1}(V_{i4})^{\alpha_i} - 2^{1-\beta_i} k_{i2}(V_{i4})^{\beta_i} + g_{ui}^* |e_{U_i}| + g_{ri}^* |e_{r_i}|$ , where  $k_{i1} > 0$ ,  $k_{i2} > 0$ ,  $1 > \alpha_i > 0$ , and  $\beta_i > 1$ . It follows from Lemma 2 that the errors  $|g_{ui}|$  and  $|g_{ri}|$  converge to zero within a fixed-time and  $\dot{V}_{i4} \leq -k_{i1}(V_{i4})^{\alpha_i} - 2^{1-\beta_i} k_{i3}(V_{i4})^{\beta_i}$ , where  $k_{i3} = 2^{1-\beta_i} k_{i2}$ . Thus, all the

tracking errors in the entire closed-loop system are UUB. ■

**Simulation:** Consider a multi-RSV system including six RSVs. The environment has the  $xy$  dimensions of  $100 \text{ m} \times 100 \text{ m}$  and  $\phi(q) = 1$ . The initial states of six charging station and six RSVs are set to  $p_{bi} = [2, 85 - 15(i-1)]^T$  ( $i = 1, \dots, 6$ ),  $p_1 = [28, 15]^T$ ,  $p_2 = [65, 43]^T$ ,  $p_3 = [37, 20]^T$ ,  $p_4 = [12, 15]^T$ ,  $p_5 = [55, 50]^T$ ,  $p_6 = [51, 12]^T$ ,  $\psi_{b1} = \psi_{b6} = 0$ ,  $\psi_{b2} = \pi/2$ ,  $\psi_{b3} = -4\pi/5$ ,  $\psi_{b4} = \pi/4$ ,  $\psi_{b5} = \pi$ ,  $q_i = [0, 0]^T$ ,  $r_i = 0$ ,  $E_1 = 3600 \text{ mV}$ ,  $E_2 = 3900 \text{ mV}$ ,  $E_3 = 4100 \text{ mV}$ ,  $E_4 = 3800 \text{ mV}$ ,  $E_5 = 4000 \text{ mV}$ , and  $E_6 = 3700 \text{ mV}$ . The parameters in this case are set to  $K_{ch} = 10$ ,  $K_{en}$ ,  $k = 0.02$ ,  $d_{ch}$ ,  $c = 0.5$ ,  $E_{\max} = 4200 \text{ mV}$ , and  $E_{\min} = 3000 \text{ mV}$ ,  $k_{i1}^u = k_{i2}^u = k_{i1}^r = k_{i2}^r = 10$ ,  $k_{i3}^u = k_{i3}^r = 2.5$ ,  $k_{i4}^u = k_{i4}^r = k_{i1}^r = k_{i2}^r = 20$ ,  $k_{i3}^u = k_{i4}^u = k_{i3}^r = k_{i4}^r = 100$ ,  $k_{i0}^u = k_{i0}^r = 0.05$ ,  $\alpha_{ui} = \alpha_{ri} = 0.9$ , and  $\beta_{ui} = \beta_{ri} = 1.1$ ,  $K_{qi} = \text{diag}\{1, 1\}$ ,  $k_{ri} = 0.5$ ,  $k_{i1}^r = k_{i2}^r = k_{i3}^r = k_{i4}^r = 1$ ,  $\alpha_{ui}^r = \alpha_{ri}^r = 0.9$ ,  $\beta_{ui}^r = \beta_{ri}^r = 1.1$ , and  $\Delta_{qi} = \Delta_{ri} = 1$ . The long duration coverage behavior and the energy change curve of the RSV battery are depicted in Fig. 1. It shows that the optimal coverage of the given coverage area is achieved for the first time at 125 s. At 220 s, the red RSV returns to the charging station for charging, and other RSVs continue the long duration coverage task. Next, the red RSV leaves the charging station to perform the coverage task. Then, at 630 s, the RSVs complete the optimal coverage for the second time.

**Conclusion:** In this letter, an anti-disturbance energy-aware control method for RSVs is proposed based on a CVT, two BE-CBFs,

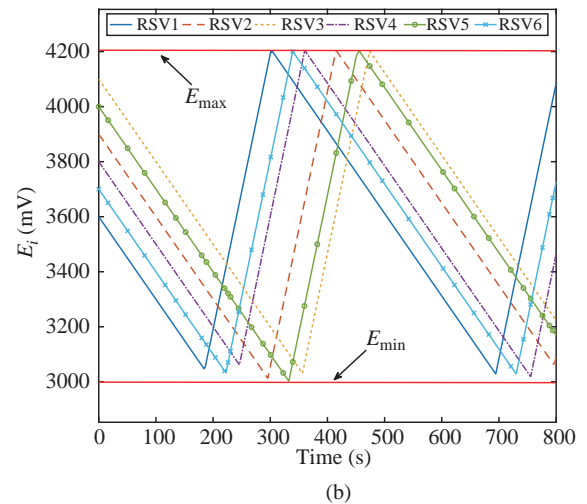
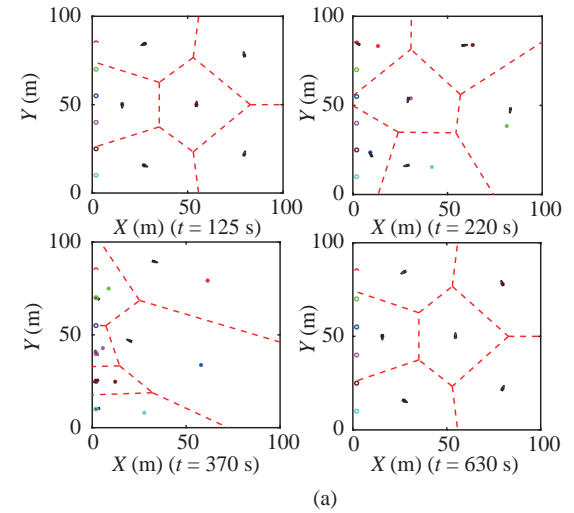


Fig. 1. Simulation results. (a) The coverage behavior; (b) The energy change curve.

and FTESO to achieve long duration coverage task. By the control method, RSVs are capable of achieving the long duration coverage within the given coverage area.

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## References

- [1] H. Li, M. Zhou, K. Hao, and E. Hou, "A survey of multi-robot regular and adversarial patrolling," *IEEE/CAA J. Autom. Sinica*, vol. 6, no. 4, pp. 894–903, 2019.
- [2] C. Jie, S. Jian, and G. Wang, "From unmanned systems to autonomous intelligent systems," *Engineering*, vol. 12, no. 5, pp. 16–19, 2022.
- [3] T. Li, R. Zhao, C. Chen, L. Fang, and C. Liu, "Finite-time formation control of under-actuated ships using nonlinear sliding mode control," *IEEE Trans. Cybern.*, vol. 48, no. 11, pp. 3243–3253, 2018.
- [4] L. Ma, Y.-L. Wang, and Q.-L. Han, "Cooperative target tracking of multiple autonomous surface vehicles under switching interaction topologies," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 3, pp. 673–684, 2023.
- [5] N. Gu, D. Wang, Z. Peng, and J. Wang, "Safety-critical containment maneuvering of underactuated autonomous surface vehicles based on neurodynamic optimization with control barrier functions," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 34, no. 6, pp. 2882–2895, 2023.
- [6] A. Kwok and S. Martinez, "Unicycle coverage control via hybrid modeling," *IEEE Trans. Automat. Contr.*, vol. 55, no. 2, pp. 528–532, 2010.
- [7] J. Li, Y. Xiong, J. She, and M. Wu, "A path planning method for sweep coverage with multiple UAVs," *IEEE Internet. Things J.*, vol. 7, no. 9, pp. 8967–8978, 2020.
- [8] L. Zuo, W. Yan, R. Cui, and J. Gao, "A coverage algorithm for multiple autonomous surface vehicles in flowing environments," *Int. J. Control*, vol. 14, no. 2, pp. 528–532, 2016.
- [9] C. Song, L. Liu, G. Feng, Y. Fan, and S. Xu, "Coverage control for heterogeneous mobile sensor networks with bounded position measurement errors," *Automatica*, vol. 120, p. 109118, 2016.
- [10] J. Hou, X. Zeng, G. Wang, J. Sun, and J. Chen, "Distributed momentum-based frank-wolfe algorithm for stochastic optimization," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 3, pp. 685–699, 2023.
- [11] Z. Pang, W. Luo, G.-P. Liu, and Q.-L. Han, "Observer-based incremental predictive control of networked multi-agent systems with random delays and packet dropouts," *IEEE Trans. Circuits-II*, vol. 68, no. 1, pp. 426–430, 2021.
- [12] C. Zheng, Z. Pang, J. Wang, J. Sun, G.-P. Liu, and Q.-L. Han, "Null-space-based time-varying formation control of uncertain nonlinear second-order multi-agent systems with collision avoidance," *IEEE Trans. Ind. Electron.*, vol. 70, no. 10, pp. 10476–10485, 2023.